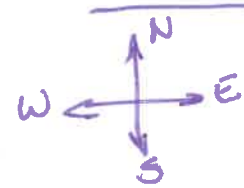
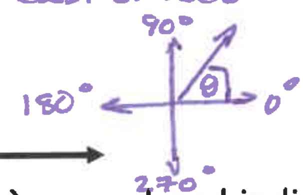
Chapter 5 - Vectors and Projectile Motion

Directions



- North/south first, and then how many degrees east or west.



Scalar Quantities - express only magnitude

ie. time, distance, speed

Vector Quantities - express magnitude and direction.

ie.	velocity	80 km/h, 58°
	displacement	10 km (E)
	acceleration	4.0 m/s^2, 27°
	force	100 N, 110°

- represented by an arrow.
- length of arrow indicates magnitude(drawn to scale), arrowhead indicates direction.

Graphical Analysis of Vectors

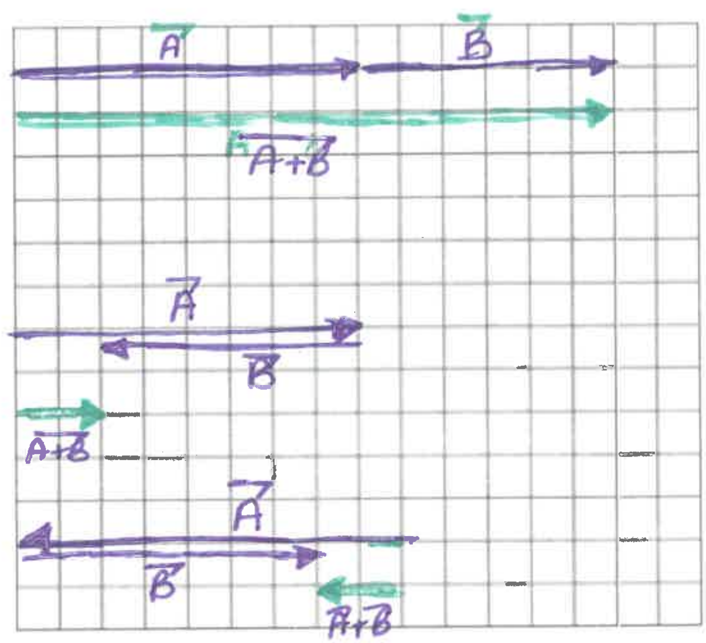
1. Add vectors by placing the tail of one vector at the head of the other vector.
2. A third vector is drawn connecting the tail of the first vector with the tip of the last vector. This vector, the resultant, represents the sum of the vectors.
3. Order of addition does not matter.

Vector Addition in 1 Dimension

Ex.1 A = 8.0 m, E
B = 6.0 m, E

Ex 2. A = 8.0 m, E
B = 6.0 m, W

Ex.3 A = 8.0 m, W
B = 6.0 m, E



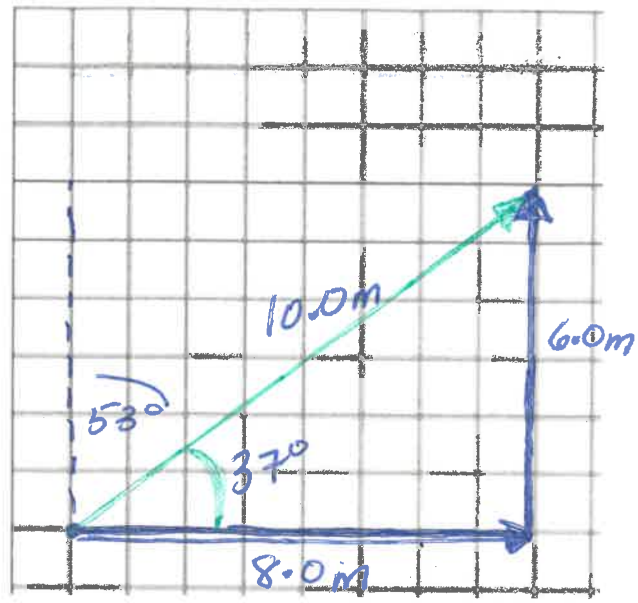
Vector Addition in 2 Dimensions

Ex. 1 A = 8.0 m E
B = 6,0 m, N

$\vec{A} + \vec{B} = \vec{C}$

$\vec{C} = 10.0\text{m} @ 37^\circ$

or $10.0\text{m} @ N53^\circ E$



Resultant

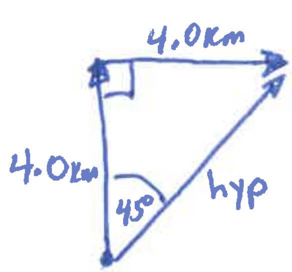
a) magnitude = measure the length from tail of the 1st vector to the tip of the last vector.

b) direction = measure the angle with a protractor from the horizontal vector, measured counterclockwise.

Several Vectors

1. A hiker travels 7 km N, then 4 km E and 3 km S. What is the hiker's final displacement?

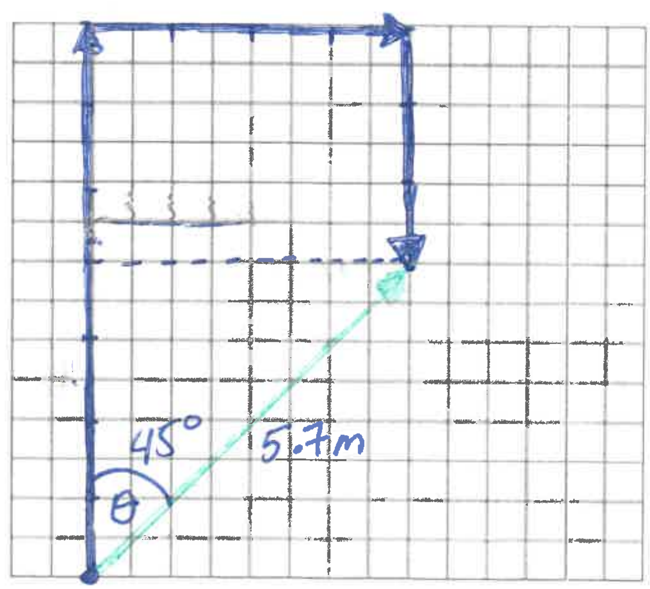
$\vec{R} = 5.7\text{km} @ N45^\circ E$



$\cos 45^\circ = \frac{4.0\text{km}}{\text{hyp}}$

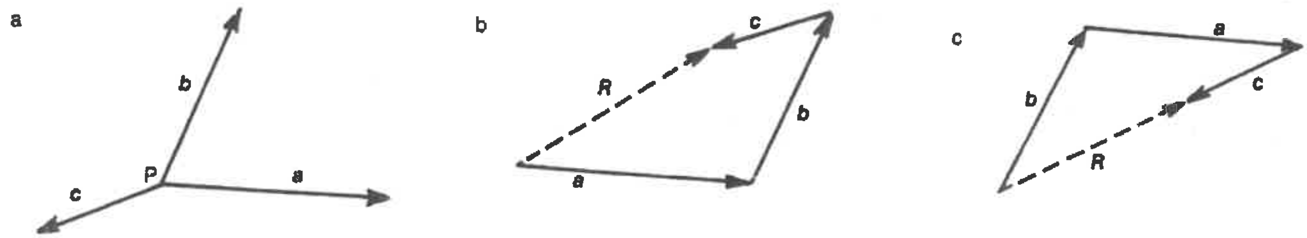
$\text{hyp} = \frac{4.0\text{km}}{\cos 45^\circ}$

$= 5.7\text{km}$



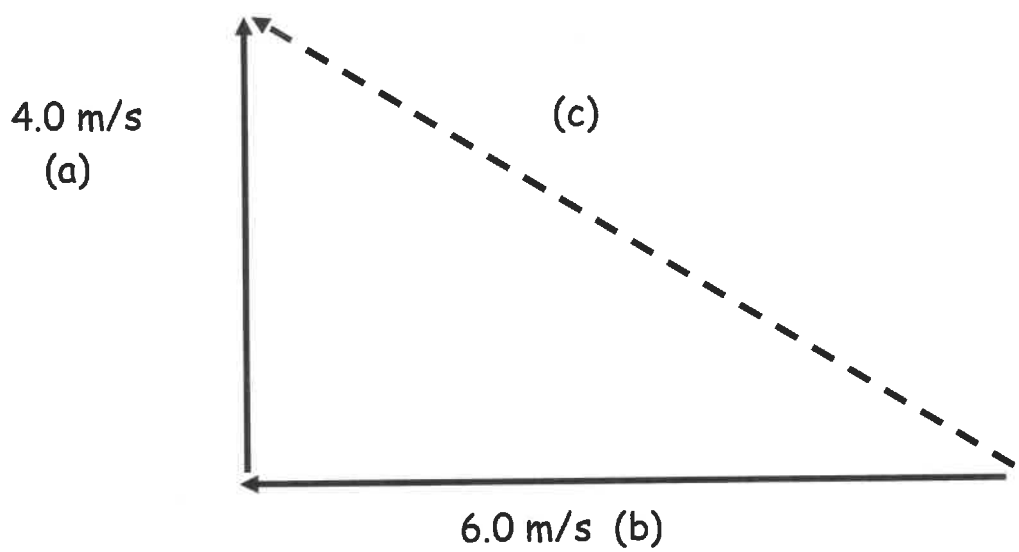
VECTOR ADDITION:

Order of addition does not matter - the resultant still has the same magnitude and direction!



Analytical Method of Vector Addition

Length of the Vector: Pythagorus Theorum



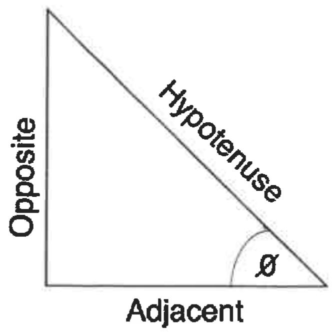
$a^2 + b^2 = c^2$

$4.0^2 + 6.0^2 = c^2$

$C = \sqrt{(4.0^2 + 6.0^2)}$

$= 7.2$ m/s

Angle of the Vector: Trigonometric Functions



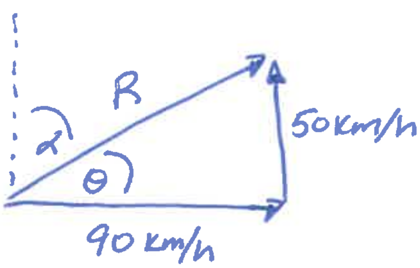
$$\sin Ø = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos Ø = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan Ø = \frac{\text{opposite}}{\text{adjacent}}$$

Remember: SOH/CAH/TOA

Ex#1: An airplane flying toward 0° at 90 km/h is being blown toward 90° at 50 km/h. What is the resultant velocity of the plane?

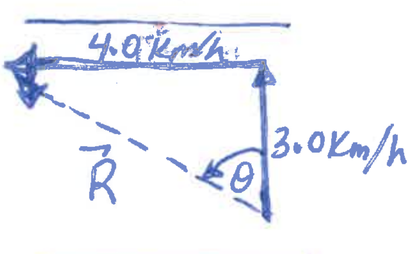


$$R = \sqrt{(50\,km/h)^2 + (90\,km/h)^2}$$
$$= 103\,km/h$$

$$\theta = \tan^{-1}\left(\frac{50\,km/h}{90\,km/h}\right) = 29°$$

$$R = 100\,km/h @ 29°$$

Ex#2. A swimmer jumps into a river and swims straight for the other side at 3.0 km/h (N). There is a current in the river of 4 km/h (W). What is the swimmers velocity relative to the shore?



$$\vec{R} = \sqrt{(4.0\,km/h)^2 + (3.0\,km/h)^2}$$
$$= 5.0\,km/h$$

$$\theta = \tan^{-1}\left(\frac{4.0\,km/h}{3.0\,km/h}\right) = 53°$$

$$\vec{R} = 5.0\,km/h @ N53°W$$

Vector Addition

1. If a vector that is 1 cm long represents a displacement of 5 m, how many metres does a vector 3 cm long, drawn to the same scale, represent?

$$\frac{1cm}{5m} \overset{\times 3}{=} \frac{3cm}{Xm} \qquad \times 3 \qquad X = 15\,m$$

2. A vector drawn 15 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?

$$\frac{15\,mm}{30\,m/s} = \frac{x}{20\,m/s} \qquad x = \frac{(15\,mm)(20\,m/s)}{30\,m/s} = 10\,mm$$

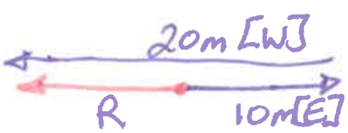
3. An airplane normally flies at 200 km/h. What is the resultant velocity of the airplane if:
   a) it experiences a 50 km/h tail wind?

   $\xrightarrow{200km/h} \xrightarrow{50km/h} = 250\ km/h$

   b) it experiences a 50 km/h head wind?

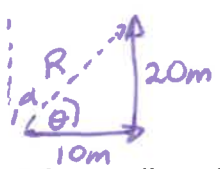
   $\xrightarrow{200km/h}$ $\xleftarrow[50\ km/h]{}$ $= 150\ km/h$

4. Find the final displacement when the following vectors are added:
   a) 10 m E and 20 m W



10m [W]

   b) 10 m E and 20 m N



$R = \sqrt{20^2 + 10^2}$
$= 22m$
$\theta = \tan^{-1}\left(\frac{20}{10}\right) = 63^\circ,\ \alpha = 27^\circ$
$\therefore R = 22m$ @ N27°E

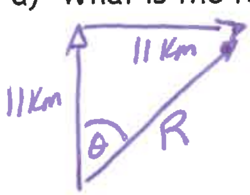
   c) 10 m E and 20 m S

$R = \sqrt{10^2 + 20^2}$
$= 22m$
$\theta = \tan^{-1}\left(\frac{20}{10}\right) = 63^\circ,\ \alpha = 27^\circ$
$\therefore R = 22m$ @ S27°E

   d) 10 m W and 20 m E

R = 10m[E]

5. After walking 11 km due north from camp, a hiker then walks 11 km due east.
   a) What is the total distance walked by the hiker?



total distance = 11 m + 11 m = 22 m

   b) Determine the total displacement from the starting point.

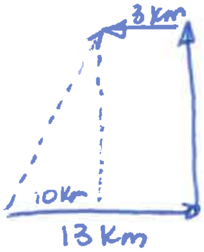
$\vec{d} = \sqrt{(11km)^2 + (11km)^2} = 16\,m \qquad \theta = \tan^{-1}\left(\frac{11km}{11km}\right) = 45^\circ$

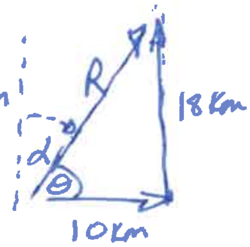
$= 16m$ @ N45°E

6. An explorer walks 13 km due east, then 18 km north, and finally 3 km west.
   a) What is the total distance walked?

$$13\text{ km} + 18\text{ km} + 3\text{ km} = 34\text{ km}$$

   b) What is the resulting displacement of the explorer from the starting point?





$$\vec{R} = \sqrt{(18\text{ km})^2 + (10\text{ km})^2} = 20.6\text{ km}$$

$$\theta = \tan^{-1}\left(\frac{18}{10}\right) = 61^\circ$$

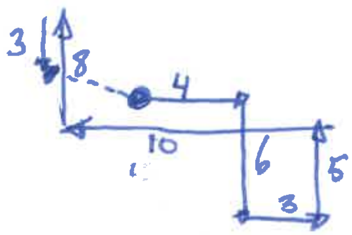
$$\alpha = 90^\circ - 61^\circ = 29^\circ$$

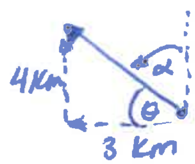
$$\therefore \vec{R} = 21\text{ km @ N}29^\circ\text{E}$$

7. A hiker leaves camp and, using a compass, walks 4 km E, 6 km S, 3km E, 5 km N, 10 km W, 8 km N, and 3 km S. At the end of three days, the hiker is lost. By drawing a diagram, compute how far the hiker is from camp and which direction should be taken to get back to camp.

Re-order: 4 km[E] + 3 km[E] + 10 km[W] = 3 km[W]

6 km[S] + 5 km[N] + 8 km[N] + 3 km[S] = 4 km[N]





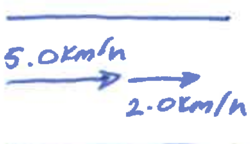
$$R = \sqrt{3^2 + 4^2} = 5\text{ km}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$$

$$\alpha = 90^\circ - 53^\circ = 37^\circ$$

$$\therefore R = 5\text{ km @ N}37^\circ\text{W}$$

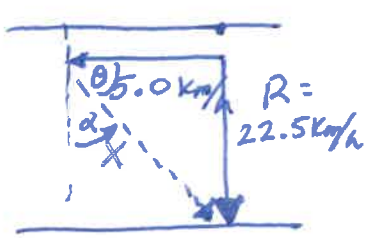
8. You head down a river in a canoe. You paddle at 5.0 km/h and the river is flowing at 2.0 km/h. What is your resultant velocity? (Give both magnitude and direction).



$$5.0\text{ km/h} + 2.0\text{ km/h} = 7.0\text{ km/h}$$

downstream

9. CHALLENGE: Kyle wishes to drive his boat across a river to a point 4.5 km due south in 12 minutes. The river is flowing westward with a current of 5.0 km/h.. Compute the proper heading and speed that Kyle must choose in order to reach his destination on time.



$$12\text{ min} \times \frac{1\text{ h}}{60\text{ min}} = 0.20\text{ h}$$

$$V_S = \frac{4.5\text{ km}}{0.20\text{ h}} = 22.5\text{ km/h}$$

$$X = \sqrt{(5.0\text{ km/h})^2 + (22.5\text{ km/h})^2} = 23.0\text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{22.5}{5.0}\right) = 77^\circ$$

$$\alpha = 90^\circ - 77^\circ = 13^\circ$$

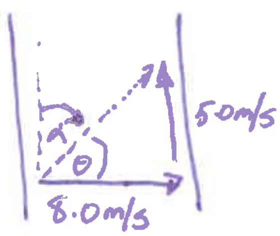
$$\therefore R = 23\text{ km/h @ S}13^\circ\text{E}$$

Independence of Perpendicular Velocity Vectors

- perpendicular vector quantities are independent of each other; if I change my velocity in the north-south direction, it does NOT affect my velocity in the east-west direction.

Ex#1: A motorboat heads east at 8.0 m/s across a river that flows north at 5.0 m/s.

a) Calculate the resultant velocity



$\vec{V}_R = \sqrt{(8.0\,m/s)^2 + (5.0\,m/s)^2}$

$= 9.4\,m/s$

$\theta = \tan^{-1}\left(\frac{5.0\,m/s}{8.0\,m/s}\right) = 32°$; $\alpha = 90° - 32° = 58°$

$\therefore \vec{V}_R = 9.4\,m/s$ @ N58°E

b) If it takes the boat 10 s to cross the river, what is the width of the river.

$V_E = 8.0\,m/s$

$t = 10s$

$d_E = ?$

$d_E = V_E \cdot t$

$= (8.0\,m/s)(10s)$

$= 80\,m$ wide.

c) How far down the river did he travel?

$V_N = 5.0\,m/s$

$t = 10s$

$d_N = ?$

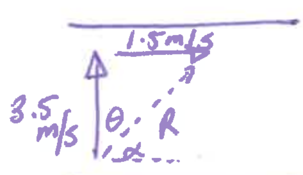
$d_N = V_N \cdot t$

$= 5.0\,m/s \cdot 10s$

$= 50\,m$ down-river

Ex#2. A boat travels 3.5 m/s and heads straight across a river that is 240m wide.

a) if the river flows at 1.5 m/s, what is the resultant speed of the boat relative to the shore?



$\vec{V}_R = \sqrt{(3.5\,m/s)^2 + (1.5\,m/s)^2}$

$= 3.8\,m/s$

$\theta = \tan^{-1}\left(\frac{1.5}{3.5}\right) = 23°$; $\alpha = 67°$

The boat moves at 3.8 m/s at 67° from shoreline

b) How long does it take the boat to cross the river?

$d_N = 240m$

$V_N = 3.5\,m/s$

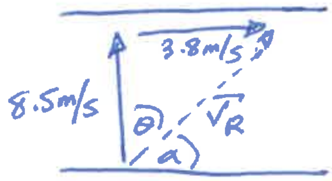
$t = ?$

$V_N = \frac{d_N}{t}$

$t = \frac{d_N}{V_N} = \frac{240m}{3.5\,m/s} = 69s$

Independence of Vectors

1. A speedboat travels at 8.5 m/s. It head straight across a river 110 m wide.
   a) If the water flows downstream at a rate of 3.8 m/s, what is the boat's resultant velocity?



$$V_R = \sqrt{(8.5\,m/s)^2 + (3.8\,m/s)^2} = 9.3\,m/s$$

$$\theta = \tan^{-1}\left(\frac{3.8\,m/s}{8.5\,m/s}\right) = 24.°;\ \alpha = 90° - 24° = 66°$$

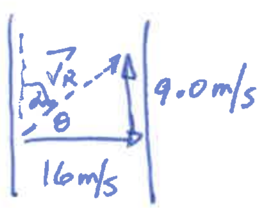
$$\therefore\ \vec{V}_R = 9.3\,m/s\ @\ 66° \text{ from shore}$$

   b) How long does it take the boat to reach the opposite shore?

$V_y = 8.5\,m/s$
$d_y = 110\,m$
$t = ?$

$$V_y = \frac{d_y}{t};\quad t = \frac{d_y}{V_y} = \frac{110\,m}{8.5\,m/s} = 13\,s$$

2. A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.
   a) What is the resultant velocity (speed and direction) of the boat?



$$\vec{V}_R = \sqrt{(9.0\,m/s)^2 + (16\,m/s)^2} = 18.4\,m/s$$

$$\theta = \tan^{-1}\left(\frac{9.0\,m/s}{16\,m/s}\right) = 29°;\ \alpha = 90° - 29° = 61°$$

$$\vec{V}_R = 18\,m/s\ @\ N61°E$$

   b) If the river is 136 m wide, how long does it take the motorboat to reach the other side?

$V_E = 16\,m/s$
$d_E = 136\,m$
$t = ?$

$$V_E = \frac{d_E}{t}$$

$$t = \frac{d_E}{V_E} = \frac{136\,m}{16\,m/s} = 8.5\,s$$

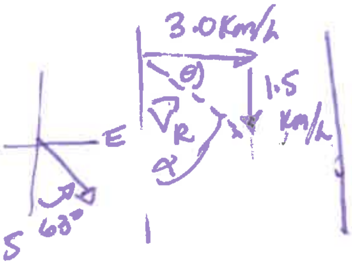
   c) How far downstream is the boat when it reaches the other side of the river?

$V_N = 9.0\,m/s$
$t = 8.5\,s$
$d_N = ?$

$$d_N = V_N \cdot t$$
$$= (9.0\,m/s)(8.5\,s)$$
$$= 77\,m\ [N]$$

3. Paul swims due east at a rate of 3.0 km/h, while the river he is crossing flows south at 1.5 km/h.

a) What is Paul's resultant velocity?



$\vec{V}_R = \sqrt{(1.5\ km/h)^2 + (3.0\ km/h)^2}$

$= 3.35\ m/s$

$\therefore \vec{V}_R = 3.4\ m/s$ @ S63°E

$\theta = \tan^{-1}\left(\frac{1.5}{3}\right) = 27°$

$\alpha = 90° - 27° = 63°$

b) If it takes Paul 20.0 minutes to cross the river, how wide is it?

$V_E = 3.0\ km/h$

$t = 0.333\ h$

$d_E = ?$

$d_E = V_E \cdot t$

$= (3.0\ km/h)(0.333\ h)$

$= 1.0\ km$

c) How far downstream will Paul reach the other side?

$V_S = 1.5\ km/h$

$t = 0.333\ h$

$d_S = ?$

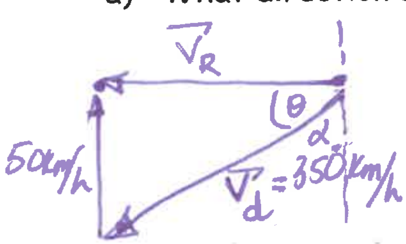
$d_S = V_S \cdot t$

$= (1.5\ km/h)(0.333\ h)$

$= 0.5\ km$ (south)

4. CHALLENGE: A pilot wants to fly to a city that is 500 km due west of his current position. The wind is blowing towards the north at 50 km/h. The maximum air speed of the airplane is 350 km/h.

a) What direction should the pilot fly in to reach the city?



$\theta = \sin^{-1}\left(\frac{50}{350}\right) = 8°$

$\alpha = 90° - 8° = 82°$

$\therefore$ S82°W

b) How long will it take the plane to reach its destination?

$V_R^2 + 50^2 = 350^2$

$V_R = \sqrt{(350\ km/h)^2 - (50\ km/h)^2}$

$= 346\ km/h$

$V_R = 346\ km/h$

$d_R = 500\ km$

$t = ?$

$t = \frac{d_R}{V_R}$

$= \frac{500\ km}{346\ km/h}$

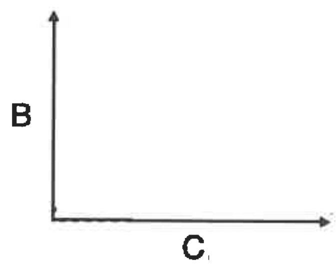
$= 1.4\ h$

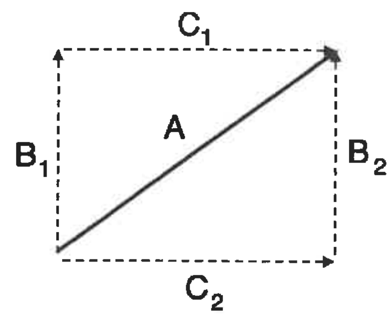
Components of Vectors

- 2 vectors acting in different directions may be replaced by a single vector the **resultant**.

Therefore:

- a single vector is the resultant of 2 vectors.

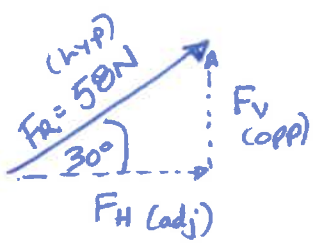




A is a result of B and C.

Vector Resolution - finding the magnitude of a component in a given direction.

Ex#1. A person pulling a sled exerts a force of 58N on a rope held at an angle of 30° with the horizontal. What is the vertical component? What is the horizontal component?



$\sin\theta = \frac{F_V}{F_R}$

$F_V = F_R \sin\theta$

$= (58\ N)(\sin 30°)$

$= 29N$

$\cos\theta = \frac{F_H}{F_R}$

$F_H = F_R \cos\theta$

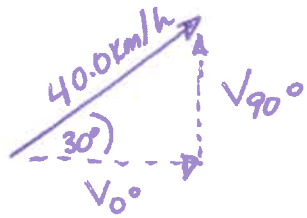
$= (58N)(\cos 30°)$

$= 50.2N$

$= 5.0 \times 10^1 N$

Ex#2. A wind with a velocity of 40.0 km/h blows towards 30.0°.

a) What is the component of the winds velocity toward 90.0°?

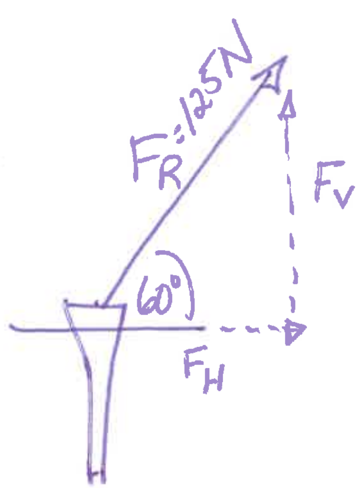


$$V_{90°} = V_R \sin 30°$$
$$= (40.0\,km/h)(\sin 30°)$$
$$= 20.0\,km/h$$

b) What is the component of the winds velocity toward 0°?

$$V_{0°} = V_R \cos 30°$$
$$= (40.0\,km/h)(\cos 30°)$$
$$= 34.6\,km/h$$

Ex#3. Beth attempts to pull a stake out of the ground by pulling a rope that is attached to the stake. The rope makes and angle of 60.0° with the horizontal. Beth exerts a force of 125N on the rope. What is the magnitude of the vertical component of the force acting on the stake.

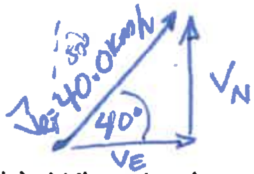


$$F_V = F_R \sin 60°$$
$$= (125\,N)\sin 60°$$
$$= 108\,N$$

Components of Vectors

1. A wind with a velocity of 40.0 km/h blows towards N50.0°E.

   a) What is the component of the wind's velocity towards the north?

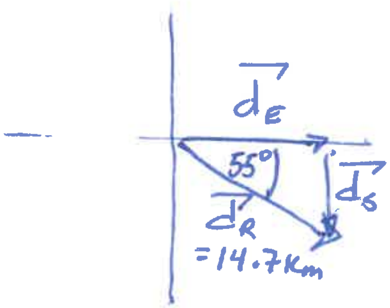


$$V_N = V_R \sin 50.0°$$
$$= (40.0\ km/h)(\sin 40.0°) = 25.7\ km/h$$

   b) What is the component of the wind's velocity towards the east?

$$V_E = V_R \cos 50.0°$$
$$= (40.0\ km/h)(\cos 50.0°) = 30.6\ km/h$$

2. A hiker walks 14.7 km at an angle of 305° from east. Find the east-west and north-south components of this walk.



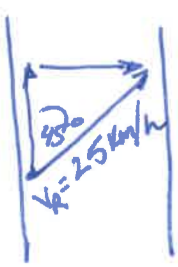
$$\vec{d_E} = \vec{d_R} \cos 55°$$
$$= (14.7\ km)(\cos 55°)$$
$$= 8.43\ km$$

$$\vec{d_S} = d_R \sin 55°$$
$$= (14.7\ km)(\sin 55°)$$
$$= 12.04\ km$$

∴ Components are: 8.4 km [E]
12.0 km [S]

3. A boat travels at a velocity of 25 km/h at a heading of N45°E. The river is 1500 m wide and flows due north.

   a) How long does it take the boat to cross the river?



$$V_E = V_R \sin \theta$$
$$= (25\ km/h)(\sin 45°)$$
$$= 17.67\ km/h$$

$V_E = 17.67\ km/h$

$d_E = 1500\ m = 1.5\ km$

$t = ?$

$$t = \frac{d_E}{V_E} = \frac{1500\ km}{17.67\ km/h}$$
$$= 0.0849\ h$$
$$\doteq 5.1\ min$$

   b) How far downstream will the boat hit the other shore?

$$V_N = V_R \cos 45°$$
$$= (25\ km/h)(\cos 45°)$$
$$= 17.65\ km/h$$

$V_N = 17.65\ km/h$

$t = 0.0849\ h$

$d_N = ?$

$$d_N = V_N t$$
$$= (17.65\ km/h)(0.0849\ h)$$
$$\doteq 1.5\ km$$

Projectile Motion

Projectiles - an object that is launched.
ie. thrown baseball, kicked football, speeding bullet.

Trajectory - the path of a projectile.

Objects Launched Vertically

If an object is thrown upwards:

- the initial vertical velocity will be have a positive magnitude.
- The vertical velocity at the top of the trajectory will be zero.
- The time it takes to get to the top of the trajectory is equal to half the time of the total trajectory.

Example: A ball is thrown straight upwards with a (initial) vertical velocity of +12 m/s.

a) How high does the ball go?

$v_i = +12 m/s$
$v_f = 0 m/s$
$a = -9.81 m/s^2$
$d = ?$

$v_f^2 = v_i^2 + 2ad$

$d = \frac{v_f^2 - v_i^2}{2a}$

$d = \frac{(0 m/s)^2 - (+12 m/s)^2}{2(-9.81 m/s^2)}$

$= 7.34 m$

$= 7.3 m$

b) What is the ball's final velocity if it is caught at the same height it was thrown? How does it compare to the initial velocity?

Negative since is falls down

$v_i = 0 m/s$
$d = -7.34 m$
$a = -9.81 m/s^2$
$v_f = ?$

$v_f^2 = v_i^2 + 2ad$

$v_f = \pm\sqrt{v_i^2 + 2ad}$

$v_f = \sqrt{(0 m/s)^2 + 2(-9.81 m/s^2)(-7.34)}$

$= -12.00 m/s$

$= -12 m/s$

Same magnitude, opposite direction

c) How long is the ball in the air?

$v_i = +12 m/s$
$v_f = -12 m/s$
$a = -9.81 m/s^2$
$t = ?$

$v_f = v_i + at$

$t = \frac{v_f - v_i}{a} = \frac{-12 m/s - +12 m/s}{-9.81 m/s^2} = 2.446 s$

$= 2.4 s$

Vertical Projectiles Practice

1. A penny is thrown upwards with an initial velocity of 4.2 m/s. How high does the penny go?

$v_i = +4.2\,m/s$
$a = -9.81\,m/s^2$
$v_f = 0\,m/s$
$d = ?$

$v_f^2 = v_i^2 + 2ad$

$d = \frac{v_f^2 - v_i^2}{2a}$

$d = \frac{(0\,m/s)^2 - (4.2\,m/s)^2}{(2)(-9.81\,m/s^2)}$

$= 0.899\,m = +0.90\,m$

2. A water balloon thrown upwards is in the air for 2.5 seconds.
   a. How high did the balloon go?

2nd half of trajectory:
$v_i = 0\,m/s$
$t = \frac{1}{2}(2.5\,s) = 1.25\,s$
$a = -9.81\,m/s^2$
$d = ?$

$d = v_i t + \frac{1}{2}at^2$
$= (0\,m/s)(1.25\,s) + (\frac{1}{2})(-9.81\,m/s^2)(1.25\,s)^2$
$= 7.664\,m$ $= 7.7\,m$

   b. With what velocity was the balloon originally thrown?

1st half of trajectory:
$v_f = 0\,m/s$
$a = -9.81\,m/s^2$
$t = 1.25\,s$
$v_i = ?$

$v_f = v_i + at$
$v_i = v_f - at$
$= 0\,m/s - (9.81\,m/s^2)(1.25\,s) = +12.26\,m/s = +12\,m/s$

3. A rocket shot straight upwards reaches a maximum height 150 m. How long is the rocket in the air for?

2nd half of trajectory:
$v_i = 0\,m/s$
$a = -9.81\,m/s^2$
$d = -150\,m$
$t = ?$

$d = \cancel{v_i t} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(-150\,m)}{-9.81\,m/s^2}} = 5.53\,s$

$\therefore$ total trajectory $= 2 \times 5.53\,s = 11\,s$

4. A pencil is thrown upwards towards the ceiling that is 2.1 m above a student's desk. What is the minimum velocity that the pencil would need to be thrown if it is to stick in the ceiling?

$v_f \geq 0\,m/s$
$a = -9.81\,m/s^2$
$d = +2.1\,m$
$v_i = ?$

$v_f^2 = v_i^2 + 2ad$

$v_i = \sqrt{v_f^2 - 2ad}$

$v_i = \pm\sqrt{(0\,m/s)^2 - (2)(-9.81\,m/s^2)(+2.1\,m)}$
$= +6.42\,m/s$
$= +6.4\,m/s$

1. 0.90 m; 2. a) 7.7 m; b) +12 m/s; 3. 11s; 4. 6.4 m/s

Objects Launched Horizontally

The vertical and horizontal velocities of a projectile are independent of each other.

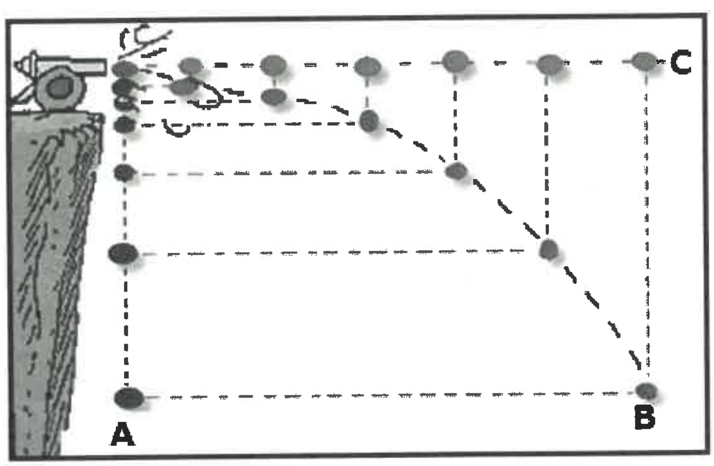
2 cannon balls:

B - launched horizontally at 20.0 m/s

A - dropped from the same height

- A has a constant horizontal velocity of 0 m/s. A falls at -9.81 m/s^2
- B moves horizontally at a constant velocity of 20.0 m/s.

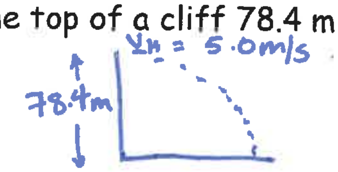
  (Newton's 1st Law - Inertia).
- B also falls at 9.81 m/s^2
- Both fall at the same rate and hit the ground at the same time.



* Shoot + Drop Video

The horizontal velocity is Constant. The vertical velocity is changing due to gravity.

Ex#1: A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff 78.4 m high.



b) How long does it take the stone to strike the ground?

Vertically

$v_{i_v} = 0\ m/s$

$a_v = -9.81 m/s^2$

$d_v = -78.4 m$

$t = ?$

$d = \cancel{v_i t} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2d_v}{a_v}} = \sqrt{\frac{(2)(-78.4m)}{-9.81 m/s^2}} = 3.998 s$

$= 4.00 s$ (3 sf)

$\boxed{t_H = t_V}$

c) How far from the base of the cliff does the stone strike the ground?

Horizontally

$v_{i_H} = 5.0 m/s$

$v_{f_H} = 5.0 m/s$

$t = 3.998 s$

(same time vertically + horizontally)

$d_H = \left(\frac{v_f + v_i}{2}\right)t$

$= (5.0 m/s)(3.998 s)$

$= 19.99 m = 2.0 \times 10^1 m$ (2 sf)

d) What is the stones vertical velocity when it hits the ground?

Vertically

$v_{i_v} = 0\ m/s$

$a_v = -9.81 m/s^2$

$d_v = -78.4 m$

$t = 3.998 s$

$v_{f_v} = v_{i_v} + at$

$= 0 m/s + (-9.81 m/s^2)(3.998 s)$

$= -39.2 m/s$

or

$v_f = \pm\sqrt{v_i^2 + 2ad}$

$= \sqrt{(0 m/s)^2 + (2)(-9.81 m/s^2)(-78.4 m)}$

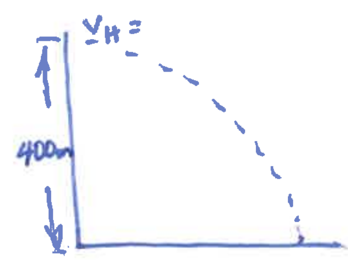
$= \pm 39.2 m/s$

$= -39.2 m/s$

Ex#2: Thelma and Louise drive off a cliff 400m high, travelling at 72km/h.

$72 km/h \times \frac{1000m}{1 km} \times \frac{1h}{3600s} = 20.0 m/s$

a) How long does it take to hit the ground?



Vertically

$v_{i_v} = 0 m/s$

$a_v = -9.81 m/s^2$

$d_v = -400m$

$d_v = \cancel{v_i t} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2d_v}{a_v}}$

$= \sqrt{\frac{(2)(-400m)}{-9.81 m/s^2}} = 9.03 s = 9.0 s$

b) How far from the base of the cliff does it hit the ground?

$v_{i_H} = 20.0 m/s$

$v_{f_H} = 20.0 m/s$

$t = 9.03 s$

$d_H = ?$

$d_H = \left(\frac{v_f + v_i}{2}\right)t$

$= (20.0 m/s)(9.03 s)$

$= 180.6 m = 180 m$ (2 sf)

Change to new Question

c) What is it's vertical velocity when it hits the ground?

$v_{i_v} = 0 m/s$

$t = 9.03 s$

$a_v = -9.81 m/s^2$

$v_{f_v} = ?$

$v_f = v_i + at$

$= 0 m/s + (-9.81 m/s^2)(9.03 s) = -88.6 m/s$

$= -89 m/s$

Projectiles Practice

1. Consider the ball's trajectory .

   a. Where is the ball travelling the fastest in the vertical direction?

      At the end of the throw

   b. Where is the ball moving the slowest in the vertical direction?

      At the top of the trajectory

   c. Where is the horizontal velocity the greatest?

      Same at all times

2. An airplane pilot flying at constant velocity and altitude drops a flare. Ignoring air resistance, where will the plane be relative to the flare when the flare hits the ground?

   Directly above the flare; the flare's horizontal velocity stays constant and will continue to travel forward as it drops.

3. A zoologist standing on a cliff aims a tranquilizer gun at a monkey hanging from a distant tree branch. The barrel of the gun is horizontal. Just as the zoologist pulls the trigger, the monkey lets go of the branch and begins to fall. Will the dart hit the monkey?

   Yes. The dart will accelerate downwards at the same rate as the monkey.

4. A stone is thrown horizontally at a speed of +5.0 m/s from the top of a cliff 78.4 m high.

   a. How long does it take the stone to reach the bottom of the cliff?

   Vertically

   $v_{i_v} = 0\,m/s$

   $a_v = -9.81\,m/s^2$

   $d_v = -78.4\,m$

   $d_v = v_i \cancel{t} + \frac{1}{2}at^2$

   $t = \sqrt{2d/a}$

   $= \sqrt{\frac{2(-78.4m)}{-9.81m/s^2}}$

   See previous Page (same Q!)

   b. How far from the base of the cliff does the stone strike the ground?

   $v_H = +5.0\,m/s$

   $t =$

   Same Q.

   c. What are the horizontal and vertical components of the velocity of the stone just before it hits the ground?

5. How would the three answers to Question #4 change if:

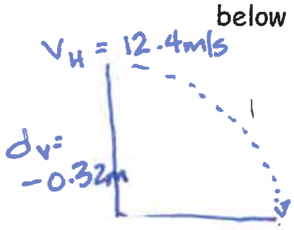
   a. The stone was thrown with twice the horizontal speed

   - The time wouldn't change
   - The distance horizontally travelled would double
   - The final vertical v will stay the same
   - Final horizontal velocity doubles (does not change from initial, though)

   b. The stone was thrown with the same speed but the cliff was twice as high?

   - The time would increase
   - distance horizontally will increase.
   - The final vertical velocity will increase
   - Final horizontal velocity stays the same

6. A dart player throws a dart horizontally at a speed of 12.4 m/s. The dart hits the board 0.32 m below the height from which it was thrown. How far away is the player from the board?



Vertically

$v_i = 0 m/s$

$d_v = -0.32 m$

$a = -9.81 m/s^2$

$t = ?$

$d_v = \cancel{v_i t} + \frac{1}{2} a_v t^2$

$t = \sqrt{\frac{2d}{a}}$

$= \sqrt{\frac{(2)(-0.32m)}{-9.81 m/s^2}} = 0.2554 s$

Horizontally:

$v_H = 12.4 m/s$

$t_v = 0.2554 s$

$d_H = ?$

$d_H = v_H t$

$= (12.4 m/s)(0.2554s)$

$= 3.17 m$

$= 3.2m$

7. An automobile, moving too fast on a horizontal stretch of mountain road, slides off the road, falling into deep snow 43.9 m below the road and 87.7 m beyond the edge of the road.

   a. How long did the auto take to fall?

   vertically: $v_i = 0 m/s$

   $d = -43.9 m$

   $a = -9.81 m/s^2$

   $d = \cancel{v_i t} + \frac{1}{2} a t^2$

   $t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{(2)(-43.9m)}{-9.81 m/s^2}} = 2.992 s$

   $= 2.99 s$

   b. How fast was it going when it left the road?

   $t = 2.992 s$

   $d_H = 87.7 m$

   $v_H = ?$

   $v_H = \frac{d_H}{t} = \frac{87.7 m}{2.992 s}$

   $= 29.31 m/s$ or $29.3 m/s$

   c. What was the acceleration 10 m below the edge of the road?

   $-9.81 m/s^2$ (in freefall)

1. a) At the end; b) at the top of the trajectory; c) same throughout 2. Directly overhead 3. Yes 4.a) 4.00 s; b) 20 m; c) v_x = 5.0 m/s; v_y= -39.2 m/s 5a) no change to time or vertical component of v_f; horizontal distance and horizontal v_f doubles, b) time, vertical v_f and horizontal distance increase; 6. 3.2m 7. a) 2.99 s; b) 29.3 m/s; c) -9.81 m/s^2

Chapter 6: Work and Mechanical Energy

Energy is the capacity to do work

Work

- the product of force exerted on an object and the distance the object moves in the direction of the force.

$$\boxed{W = Fd}$$

W = Work (Joules, J)
F = force applied (N)
d = distance (m)

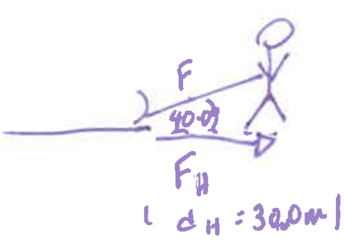
- If a large force is applied with no movement, then no work is done.
- If a force is exerted perpendicular to the motion, then no work is done.
- If a force is exerted at an angle to the motion, then the force can be replaced by its components. Only the component in the direction of motion does work.

Ex#1: How much work do you do when you climb a 3.0 m high staircase? (Assume your weight is 600 N)

$d = 3.0\text{m}$
$F_A = 600\text{N}$
$W = ?$

$W = F \cdot d$
$= (600\text{N})(3.0\text{m})$
$= 1800\text{J} = 2000\text{J}$ (1 sf)

Ex 2: A man pulls a toboggan along the snow with the rope at an angle of 40.0° with the horizontal. How much work is done by the man if he exerts a force of 255 N on the rope and pulls the toboggan 30.0 m?



$\cos 40.0° = \frac{F_H}{F}$

$F_H = 255\text{N}(\cos 40.0°)$

$W = F_H \cdot d$
$= (255\text{N})(\cos 40.0°)(30.0\text{m})$
$= 5860.2\text{ J}$
$= 5860\text{ J}$

Vectors Review

Concept Review

1. How is the resultant force affected when force vectors are added in a different order?

2. A vector drawn 16 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?

3. If a vector that is 1 cm long represents a force of 5 N, how many newtons does a vector 3 cm long, drawn to the same scale, represent?

4. A right angle triangle has two sides, A and B. If tan θ = A/B,
   a) which side of the triangle is longer if tan θ is greater than one?
   b) Which side of the triangle is longer if tan θ is less than one?

Vector Addition

1. You walk 30 m south and 28 m east. Calculate the resultant vector.

2. You head downstream on a river in a canoe. You can paddle at 5.0 km/h and the river is flowing at 2.0 km/h. How far downstream will you be in 3.0 minutes?

3. Shawn is walking at 2.5 m/s (S) on the deck of a cruise ship about to head north to Alaska. If the cruise ship starts accelerating at 3.0 m/s^2 (N), what is Shawn's velocity after 5.0 s, relative to the shore?

4. A ship leaves its home port expecting to travel to a port 500 km due south. Before it can move, a severe storm comes up and blows the ship 100 km due east. How far is the ship from its destination? In what direction must the ship travel to reach its destination?

5. Dave rows a boat across a river at 4.0 m/s. The river flows at 6.0 m/s and is 360 m across.
   a) In what direction, relative to the shore, does Dave's boat go?
   b) How long does it take Dave to cross the river?
   c) How far downstream is Dave's landing point?
   d) How long would it take Dave to cross the river if there were no current?

6. Beth, a construction worker, attempts to pull a stake out of the ground by pulling on a rope that is attached to the stake. The rope makes an angle of 60.0° to the horizontal. Beth exerts a force of 125 N on the rope. What is the magnitude of the upward component of the force acting on the stake?

Projectiles

1. A stone is thrown horizontally at a speed of 15.0 m/s from the top of a cliff 80.5 m high.
   a) how long does it take the stone to reach the bottom of the cliff?
   b) How far from the base of the cliff does the stone strike the ground?
   c) What are the horizontal and vertical components of the velocity of the stone just before it hits the ground?

2. A toy car runs off the edge of a table that is 1.225 m high. If the car lands 0.400 m from the base of the table,
   a) how long does it take for the car to fall?
   b) What is the horizontal velocity of the car?

3. A steel ball rolls with constant velocity across a tabletop 0.950 m high. It rolls off and hits the ground 0.352 m horizontally from the edge of the table. How fast was the ball rolling?

4. An automobile moving too fast on a horizontal stretch of mountain road, slides off the road, falling into deep snow 43.9 m below the road and 87.7 m beyond the edge of the road.
   a) How long did it take for the automobile to fall?
   b) How fast was it going when it left the road?
   c) What was the acceleration 10 m below the edge of the road?

5. Janet jumps off a high-diving platform with a horizontal velocity of 2.8 m/s and lands in the water 2.6 s later.
   a) How high was the platform?
   b) How far from the base of the platform does she land?

6. A dart player throws a dart horizontally at a speed of 12.4 m/s. The dart hits the board 0.320 m below the height from which it was thrown. How far away is the player from the board?

Answers:

Concept Review:

1. No change 2. 10 m/s (11 m/s) 3. 15 N 4. a) A b) B

Vector Addition:

1. 41 m at S43°E 2. 0.35 km 3. 13 m/s (N) 4. S11°W 5. a) 34° b) 9.0×10^{1}s c) 540 m d) 90 s (same amount of time) 6. 108 N

Projectiles:

1. a) 4.05 s; b) 60.8 m; c) v_H=15.0 m/s; v_V = 39.7 m/s 2. a) 0.500 s; b) 0.800 m/s 3. 0.800 m/s 4. a) 2.99s; b) 29.3 m/s; c) -9.8 m/s; 5. a) 33m; b) 7.3 m 6. 3.17 m