

## Chapter 2: Kinematics

- branch of Physics that deals with describing motion.
- motion can be described in 3 ways:
  - a) Words/sentences
  - b) mathematical equations
  - c) graphically

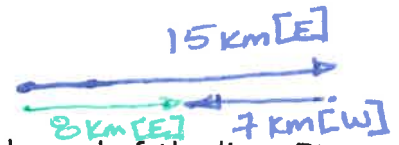
### 2.1 Displacement and Velocity

#### Distance

- the separation between 2 points.
- expresses a quantity of measure (scalar quantity).
- distances travelled in 1 direction.

#### Displacement

- the separation between an object and a reference point.
- indicates distance and direction (vector quantity).
- Used when distance traveled is not equal to distance from start.
- **reference point** - zero point used to describe motion in a frame of reference.



Ex# 1: A train leaves the main station and travels 15 km east to the end of the line. It then reverses and travels 7 km west back towards the main station.

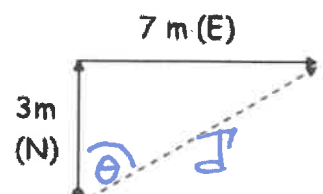
- a) What distance did the train travel? 22 km
- b) What is the train's displacement relative to the main station? 8 km [E]

Ex# 2: A football player runs 3 m north and then 7 m towards the east.

- a) What distance did the player travel? 10 m
- b) What is the player's displacement relative to his starting position?

$$\begin{aligned} \vec{d} &= \sqrt{(3\text{m})^2 + (7\text{m})^2} \\ &= \sqrt{9\text{m}^2 + 49\text{m}^2} \\ &= \sqrt{58\text{m}^2} \\ &= 7.6\text{m} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) \\ &= \tan^{-1}\left(\frac{7}{3}\right) = 67^\circ \\ \therefore \vec{d} &= 7.6\text{m} @ \text{N}67^\circ\text{E} \end{aligned}$$



## Speed

- The distance an object travels in a unit of time
- Instantaneous speed is how fast your going at an instant of time (i.e., speedometer)
- Average speed is the total distance traveled over total time.

$$\bar{v} = \frac{d}{t}$$

$\bar{v}$  = average speed (m/s)

$\Delta d$  = total distance travelled (m)

$\Delta t$  = total time taken for the motion (s)

**Ex#1:** A car travels 25km in 0.70 h, then travels 35 km in the next 1.50 h. What is the average speed of the entire trip?

$$\begin{aligned}d &= 25\text{km} + 35\text{km} = 60\text{km} \\t &= 0.70\text{h} + 1.50\text{h} = 2.20\text{h} \\ \bar{v} &= ?\end{aligned}\quad \bar{v} = \frac{d}{t} = \frac{60\text{km}}{2.20\text{h}} = 27.27\text{ km/h} = \underline{27\text{ km/h}} \text{ (2 s.f.)}$$

**Ex#2:** How far will a woman travel in 15 minutes if she is driving her car down the highway at 24 m/s?

$$\begin{aligned}t &= 15\text{min} \times \frac{60\text{s}}{1\text{min}} = 900\text{s} \\ \bar{v} &= 24\text{m/s} \\ d &= ?\end{aligned}\quad \bar{v} = \frac{d}{t} \\ d &= \bar{v}t = (24\text{m/s})(900\text{s}) = 21600\text{m} = \underline{22000\text{m}} \text{ (2 s.f.)}$$

**Ex#3:** How long does it take a girl to travel 1800 m, if she is riding her bike at a rate of 2.50 m/s?

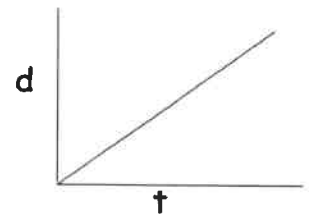
$$\begin{aligned}t &= ? \\ d &= 1800\text{m} \\ \bar{v} &= 2.50\text{m/s}\end{aligned}\quad \bar{v} = \frac{d}{t} \\ t &= \frac{d}{\bar{v}} = \frac{1800\text{m}}{2.50\text{m/s}} = \underline{720\text{s}}$$

## Velocity

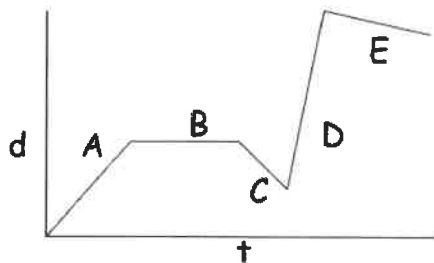
- indicates speed and direction of a moving object.
- has magnitude and direction (vector quantity)
- equal to the slope of a displacement-time graph.

## Constant Velocity

- achieved when the average velocity of an object is the same for all intervals (uniform motion).
- Constant velocities produce straight lines on displacement-time graphs



- steeper slope = greater velocity
- horizontal slope = zero velocity (stationary)
- positive velocity - indicates motion away from start point.
- negative velocity - indicates motion towards start point.

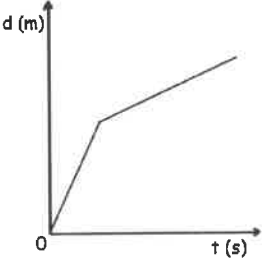
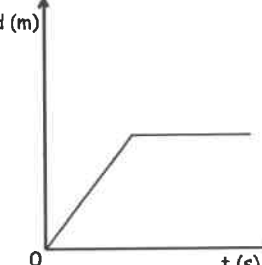
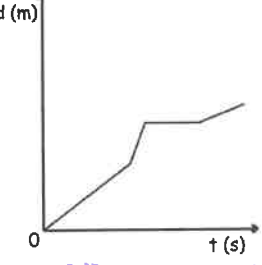
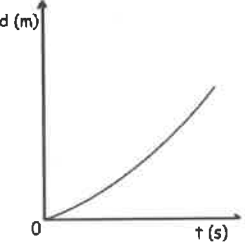
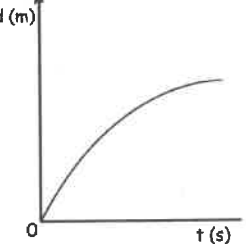
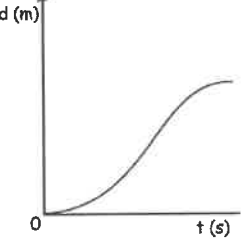
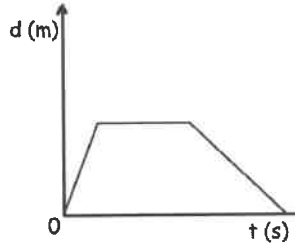
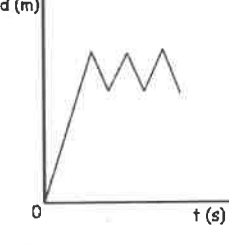
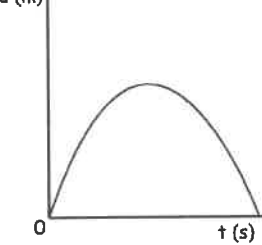


$$\begin{aligned}v_A &= \underline{\text{positive velocity}} \\v_B &= \underline{\text{zero velocity}} \\v_C &= \underline{\text{negative velocity}}\end{aligned}$$

$$\begin{aligned}v_D &= \underline{\text{fastest (positive) velocity}} \\v_E &= \underline{\text{slowest (negative) velocity}}\end{aligned}$$

## Displacement-Time Graphs

Describe the kind of motion that is taking place in each of these displacement-time graphs with respect to the starting point. If the velocity is changing, state whether it is increasing or decreasing.

<p>a)</p>  <p>- fast positive velocity suddenly changing to a slower positive velocity</p>	<p>b)</p>  <p>- positive velocity until suddenly stops</p>	<p>c)</p>  <p>- positive velocity, suddenly speeds up for a short time, then stops. After a short time, starts moving again at a slower rate away from start.</p>
<p>d)</p>  <p>Gradually speeding up away from start</p>	<p>e)</p>  <p>Gradually slowing down as object moves away from start</p>	<p>f)</p>  <p>Gradually speeds up and then gradually slows down to a stop.</p>
<p>g)</p>  <p>Moves away from start quickly at constant speed suddenly stops for a while Moves back to start at a constant speed.</p>	<p>h)</p>  <p>- fast positive constant velocity. - Reverses direction 5 times, ending moving back towards start.</p>	<p>i)</p>  <p>- Gradually slowing down as object moves away from start. Eventually stops &amp; reverses, going faster gradually back to the start</p>

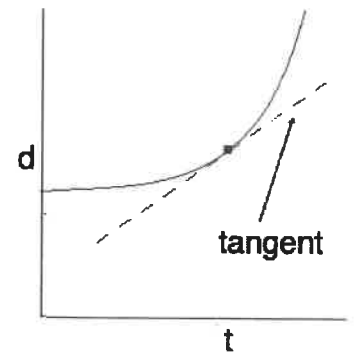
## Slope of Changing Velocity

- when velocity changes the object is accelerating
- the line on a displacement-time graph will be curved.

ie. displacement-time graph

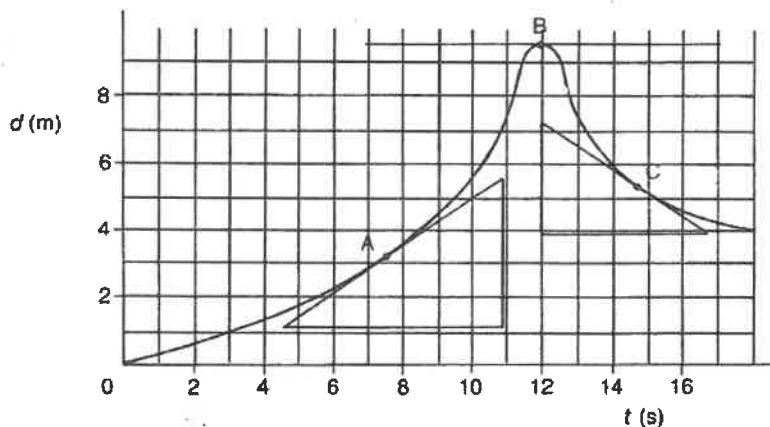
tangent - straight line that has the same slope as a point on a curve.

- Draw a line through a single point, but do not cross the graph - just touch the point.
- The velocity at that point (at one instant) is equal to the slope of the tangent.



### Sample problem

On the following displacement-time graph, find the velocity at points A, B, and C by finding the slope of the tangent to the graph at each of the points.



$$v_A = \frac{+5.4\text{m} - +1.2\text{m}}{10.9\text{s} - 4.5\text{s}}$$

$$= \frac{+4.2\text{m}}{6.4\text{s}}$$

$$= +0.66\text{m/s}$$

$$v_B = \frac{+8.5\text{m} - 8.5\text{m}}{17.0\text{s} - 7.0\text{s}}$$

$$= \frac{0\text{m}}{10.0\text{s}}$$

$$= 0\text{m/s}$$

$$v_C = \frac{+4.0\text{m} - +7.2\text{m}}{16.7\text{s} - 12.0\text{s}}$$

$$= \frac{-3.2\text{m}}{4.7\text{s}}$$

$$= -0.68\text{m/s}$$

## 2.2 Acceleration

- the rate at which velocity changes (increases or decreases).
- changing velocity = acceleration
- constant velocity = zero acceleration

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

$$a = \text{average acceleration (m/s}^2\text{)}$$

$$\Delta v = \text{change in velocity (m/s)}$$

$$\Delta t = \text{time taken for change (s)}$$

*NOTE: The  $\Delta$  symbol can be read as 'change in' the variable after it.*

**Ex#1:** A runner accelerates from 0.52 m/s to 0.78 m/s in 0.050 s. What is her acceleration?

$$v_i = +0.52 \text{ m/s}$$

$$v_f = +0.78 \text{ m/s}$$

$$\Delta t = 0.050 \text{ s}$$

$$a = ?$$

$$a = \frac{v_f - v_i}{\Delta t}$$

$$= \frac{+0.78 \text{ m/s} - +0.52 \text{ m/s}}{0.050 \text{ s}} = \frac{+0.26 \text{ m/s}}{0.050 \text{ s}} = +5.2 \text{ m/s}^2$$

**Ex#2:** A car accelerates from rest at a rate of 50.0 cm/s<sup>2</sup> for 12.5 s. How fast is it now moving?

$$v_i = 0 \text{ cm/s}$$

$$v_f = ?$$

$$a = 50.0 \text{ cm/s}^2$$

$$\Delta t = 12.5 \text{ s}$$

$$a = \frac{v_f - v_i}{\Delta t}$$

$$a \Delta t = v_f - v_i$$

$$v_f = v_i + a \Delta t$$

$$v_f = 0 \text{ cm/s} + (50.0 \text{ cm/s}^2)(12.5 \text{ s})$$

$$= 625 \text{ cm/s}$$

$$= 6.25 \times 10^2 \text{ cm/s}$$

**Ex#3:** A turtle wants to accelerate from 2 mm/s to 8 mm/s. How long will it take, if its maximum acceleration is 3 mm/s<sup>2</sup>?

$$v_i = 2 \text{ mm/s}$$

$$v_f = 8 \text{ mm/s}$$

$$a = 3 \text{ mm/s}^2$$

$$t = ?$$

$$a = \frac{v_f - v_i}{t}$$

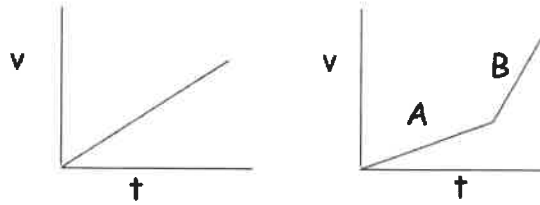
$$t = \frac{v_f - v_i}{a}$$

$$= \frac{8 \text{ mm/s} - 2 \text{ mm/s}}{3 \text{ mm/s}^2}$$

$$= \frac{6 \text{ mm/s}}{3 \text{ mm/s}^2} = 2 \text{ s}$$

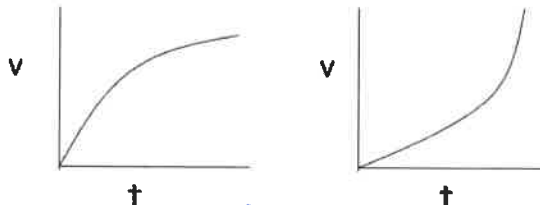
# Velocity-Time Graphs

- acceleration is equal to the slope of a velocity-time graph.



## Positive Acceleration

- constant acceleration produces a straight line (increase in velocity is the same for each unit of time).
- changing ~~velocity~~<sup>acceleration</sup> produces a curve (increase in velocity is not the same for each unit).

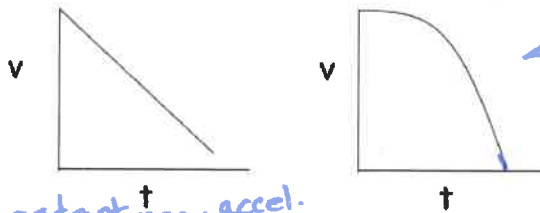


## Negative Acceleration

slowing down in +ve direction

speeding up in +ve direction

- slowing down produces negative acceleration
- straight line when decrease in velocity is the same for each unit of time.
- curved line when decrease in velocity changes with time

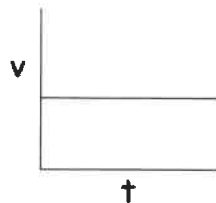


← constant velocity at start, then gradually slowing down, stopping at the end.

## Zero Acceleration

constant neg. accel.

- velocity remains constant



\* Not stopped \*

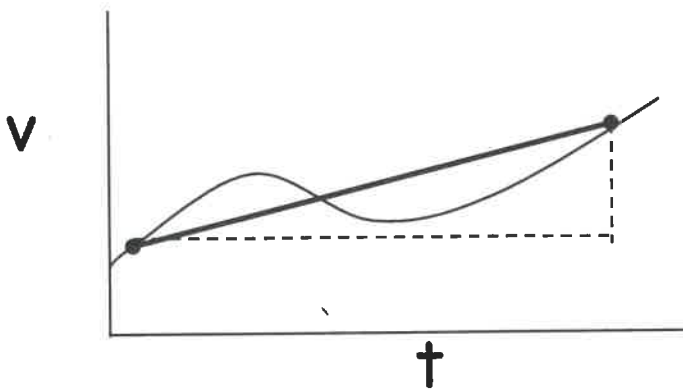
## Change in Direction

- if direction changes, the velocity changes and therefore, there is acceleration

*ie. Ferris wheel rotates at a constant speed in a circular motion, but the direction changes, resulting in acceleration.*

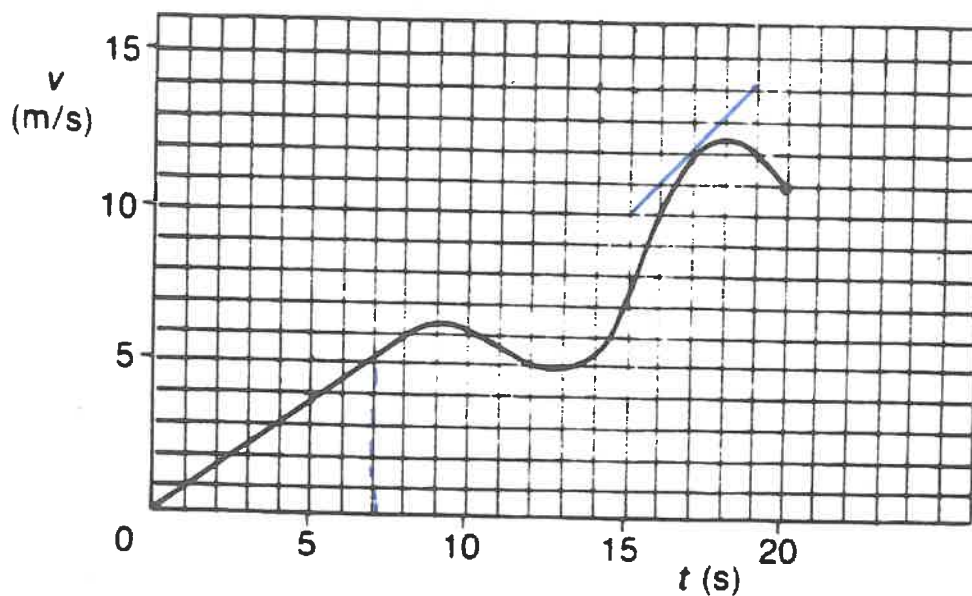
### Changing Acceleration

- curve on a V-T graph
- average acceleration = slope of the straight line joining two points on a curve of a V-T graph.



- instantaneous acceleration = slope of the tangent of a point on the graph.

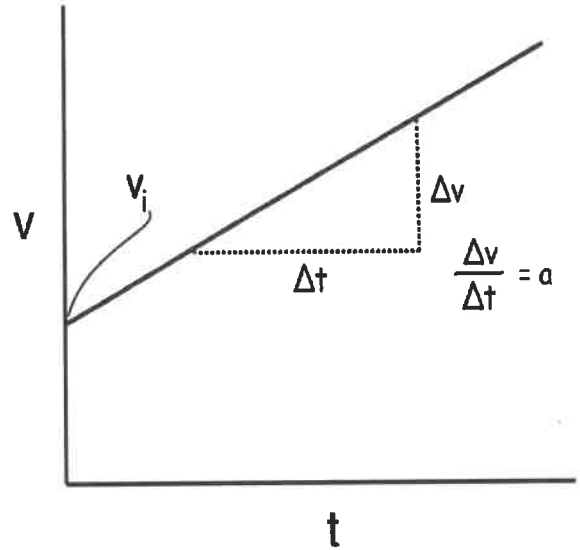




1. At what times is the acceleration zero?  $9.0\text{s}, 13.0\text{s}, 18.0\text{s}$
2. What is the acceleration for the first 7.0 s?  $\frac{5.2\text{ m/s} - 0\text{ m/s}}{7.0\text{ s}} = 0.74\text{ m/s}^2$
3. What is the average acceleration for each of the following time intervals:
  - a) 5.0 to 15.0 s  $\frac{+7.5\text{ m/s} - +3.9\text{ m/s}}{10.0\text{ s}} = +0.36\text{ m/s}^2$
  - b) 9.0 to 13.0 s  $\frac{+5.0\text{ m/s} - +6.3\text{ m/s}}{4.0\text{ s}} = -0.33\text{ m/s}^2$
  - c) 15.0 to 20.0 s  $\frac{+11.0\text{ m/s} - +7.5\text{ m/s}}{5.0\text{ s}} = +0.70\text{ m/s}^2$
4. What is the acceleration at each of the following times:
  - a) 15.0 s Pts:  $(13.0\text{s}, 3.0\text{ m/s})$   
 $(16.0\text{s}, 10.0\text{ m/s})$   
 $a = \frac{10.0\text{ m/s} - 3.0\text{ m/s}}{16.0\text{ s} - 13.0\text{ s}} = +2.3\text{ m/s}^2$
  - b) 11.0s Pts:  $(6.0\text{s}, 8.8\text{ m/s})$   
 $(15.0\text{s}, 3.0\text{ s})$   
 $a = \frac{3.0\text{ m/s} - 8.8\text{ m/s}}{15.0\text{ s} - 6.0\text{ s}} = -0.64\text{ m/s}^2$
  - c) 17.0s Pts:  $(15.0\text{s}, 10.0\text{ m/s})$   
 $(19.0\text{s}, 14.0\text{ m/s})$   
 $a = \frac{14.0\text{ m/s} - 10.0\text{ m/s}}{19.0\text{ s} - 15.0\text{ s}} = +1.0\text{ m/s}^2$

## 2.3 Uniform Acceleration

- a velocity-time graph of an object with uniform acceleration would produce a straight line
- the general equation for a line is  $y = kx + b$ , where  $k$  is the slope and  $b$  is the  $y$ -intercept.
- In the following velocity-time graph,
  - $y = v$
  - $x = t$
  - $k = \text{slope} = \Delta v / \Delta t = a$
  - $b = \text{initial velocity, or } v_i$



Therefore:

$$y = kx + b$$

$$v = at + v_i$$

$$v_f = v_i + at$$

$v_f =$  final velocity (m/s)

$v_i =$  initial velocity (m/s)

$a =$  acceleration (m/s<sup>2</sup>)

$t =$  time (s)

Ex. For the adjacent graph, determine:

a) the acceleration

Pts: (0.0s, 6.0m/s)       $a = \frac{14.0\text{m/s} - 6.0\text{m/s}}{16.0\text{s} - 0.0\text{s}}$   
 (16.0s, 14.0m/s)       $= +0.50\text{ m/s}^2$

b) the specific equation for the graph

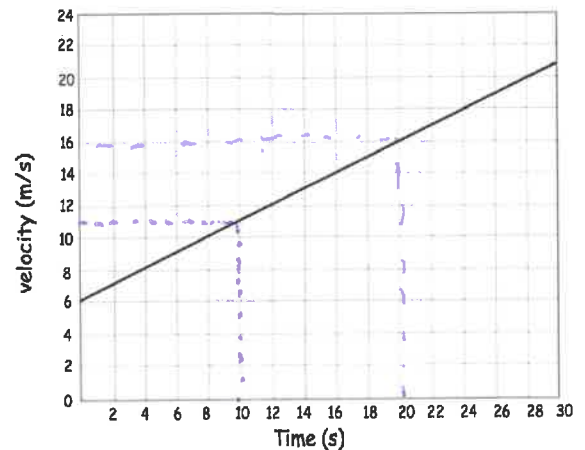
$$v_f = 6.0\text{m/s} + (+0.50\text{m/s}^2)t$$

c) the time at which  $v = 16\text{m/s}$

20.0m/s

d) the velocity at  $t = 10\text{ s}$

11.0 m/s

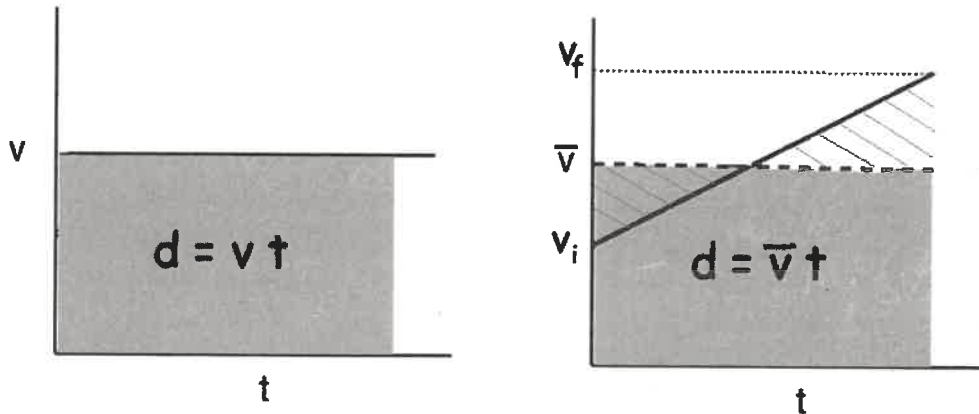


## Displacement: Given Velocity and Time

- If we know the average velocity of an object, then we can determine the distance it travels using the equation:

$$\bar{v} = \frac{d}{t} \quad \text{or} \quad d = \bar{v} t$$

- This is equal to the area under the line on a v-t graph.



- If acceleration is constant, then the average velocity can be determined using the formula:

$$\bar{v} = \frac{v_f + v_i}{2}$$

And..

$$d = \left(\frac{v_f + v_i}{2}\right) t \quad \text{or} \quad d = \frac{1}{2}(v_f + v_i)t$$

**Ex#1:** How far does a dragster travel in 6.00 s, accelerating steadily from zero to 90.0m/s?

$$\begin{aligned}
 d &= ? \\
 v_i &= 0 \text{ m/s} \\
 v_f &= 90.0 \text{ m/s} \\
 t &= 6.00 \text{ s} \\
 d &= \left(\frac{v_f + v_i}{2}\right) t \\
 &= \frac{(90.0 \text{ m/s} + 0 \text{ m/s})(6.00 \text{ s})}{2} \\
 &= (45 \text{ m/s})(6.00 \text{ s}) = 270 \text{ m} = 2.70 \times 10^2 \text{ m} \quad (3 \text{ s.f.})
 \end{aligned}$$

**Ex#2:** Two skateboarders accelerate steadily from 4.5 m/s to 11.5 m/s in 6.0 s. How far do they travel?

$$\begin{aligned}
 v_i &= 4.5 \text{ m/s} \\
 v_f &= 11.5 \text{ m/s} \\
 t &= 6.0 \text{ s} \\
 d &= ? \\
 d &= \left(\frac{v_f + v_i}{2}\right) t = \frac{(11.5 \text{ m/s} + 4.5 \text{ m/s})(6.0 \text{ s})}{2} \\
 &= (8.0 \text{ m/s})(6.0 \text{ s}) \\
 &= 48 \text{ m}
 \end{aligned}$$

## Displacement: Given Acceleration and Time

If  $v_f = v_i + at$

and  $d = \frac{1}{2}(v_f + v_i)t$

then  $d = \frac{1}{2}\{v_i + at + v_i\}t$

$$d = \frac{1}{2}(2v_i + at)t$$

$$d = v_i t + \frac{1}{2} at^2$$

**Ex#1:** A skier accelerates at  $1.20 \text{ m/s}^2$  down an icy slope starting from rest. How far does she get in 5.0s?

$$\begin{aligned} a &= 1.20 \text{ m/s}^2 & d &= v_i t + \frac{1}{2} at^2 \\ v_i &= 0 \text{ m/s} & &= (0 \text{ m/s})(5.0 \text{ s}) + \frac{1}{2}(1.20 \text{ m/s}^2)(5.0 \text{ s})^2 \\ t &= 5.0 \text{ s} & &= 15 \text{ m} \\ d &=? & & \end{aligned}$$

**Ex#2:** What is the acceleration of an object that accelerates steadily from rest, traveling a distance of 1500 m over 10 min 10.0s, ?

$$\begin{aligned} v_i &= 0 \text{ m/s} & d &= v_i t + \frac{1}{2} at^2 & a &= \frac{(2)(1500 \text{ m})}{610 \text{ s}^2} \\ d &= 1500 \text{ m} & d &= \frac{1}{2} at^2 & &= 0.0081 \text{ m/s}^2 \\ t &= 610 \text{ s} & a &= \frac{2d}{t^2} & & \\ a &=? & & & & \end{aligned}$$

**Ex#3:** How long does it take an airplane accelerating from rest at  $5.0 \text{ m/s}^2$  to travel 300 m?

$$\begin{aligned} v_i &= 0 \text{ m/s} & d &= v_i t + \frac{1}{2} at^2 & t &= \sqrt{\frac{(2)(300 \text{ m})}{5.0 \text{ m/s}^2}} \\ a &= 5.0 \text{ m/s}^2 & d &= \frac{1}{2} at^2 & &= 10.9 \text{ s} \\ d &= 300 \text{ m} & t &= \sqrt{\frac{2d}{a}} & &= 11 \text{ s (2 s.f.)} \\ t &=? & & & & \end{aligned}$$

## Displacement: Given Velocity + Acceleration

If  $v_f = v_i + at$  rearranges to:  $t = \frac{v_f - v_i}{a}$

and  $d = \left(\frac{v_f + v_i}{2}\right)t$

$$d = \left(\frac{v_f + v_i}{2}\right)\left(\frac{v_f - v_i}{a}\right)$$

$$d = \frac{(v_f + v_i)(v_f - v_i)}{2a}$$

$$2ad = (v_f + v_i)(v_f - v_i)$$

$$2ad = v_f^2 - v_i^2$$

$$v_f^2 = v_i^2 + 2ad$$

**Ex#1:** A bullet accelerates at  $6.8 \times 10^4 \text{ m/s}^2$  from rest as it travels the 0.80m of the rifle barrel. What velocity does the bullet have as it leaves the barrel?

$$a = 6.8 \times 10^4 \text{ m/s}^2$$

$$v_i = 0 \text{ m/s}$$

$$d = 0.80 \text{ m}$$

$$v_f = ?$$

$$v_f^2 = v_i^2 + 2ad = \sqrt{(0 \text{ m/s})^2 + (2)(6.8 \times 10^4 \text{ m/s}^2)(0.80 \text{ m})}$$

$$v_f = \sqrt{v_i^2 + 2ad} = 329.8 \text{ m/s}$$

$$= 330 \text{ m/s (2sf)}$$

**Ex#2.** A driver traveling at 95 km/h sees a deer standing on the road 150m ahead. He slams on his breaks and decelerates at a rate of  $-2.0 \text{ m/s}^2$ . Will he stop in time?

$$v_i = 95 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 26.4 \text{ m/s}$$

$$v_f = 0 \text{ m/s}$$

$$a = -2.0 \text{ m/s}^2$$

$$d = ?$$

$$v_f^2 = v_i^2 + 2ad = \frac{(0 \text{ m/s})^2 - (26.4 \text{ m/s})^2}{(2)(-2.0 \text{ m/s}^2)}$$

$$d = \frac{v_f^2 - v_i^2}{2a} = 174 \text{ m}$$

∩ No, he will not stop in time.

## Kinematics Practice

$$v_f = v_i + at$$

1. A golf ball rolls up a hill towards a mini-golf hole.

- a. If it starts with a velocity of +2.0 m/s and accelerates at a constant rate of  $-0.50 \text{ m/s}^2$ , what is its velocity after 2.0 s?

$$v_i = +2.0 \text{ m/s} \quad v_f = v_i + at$$

$$a = -0.50 \text{ m/s}^2 \quad = (+2.0 \text{ m/s}) + (-0.50 \text{ m/s}^2)(2.0 \text{ s}) = \boxed{+1.0 \text{ m/s}}$$

$$t = 2.0 \text{ s}$$

- b. If the acceleration occurs for 6.0 s, what is its final velocity?

$$v_i = +2.0 \text{ m/s} \quad v_f = v_i + at$$

$$a = -0.50 \text{ m/s}^2 \quad = +2.0 \text{ m/s} + (-0.50 \text{ m/s}^2)(6.0 \text{ s})$$

$$t = 6.0 \text{ s}$$

$$v_f = ? \quad = \boxed{-1.0 \text{ m/s}}$$

- c. Describe in words, the motion of the golf ball.

The ball slows down as it goes up the hill until it stops and then begins rolling back down the hill, going faster over time.

2. A bus traveling at +30 km/h accelerates at a constant  $+3.5 \text{ m/s}^2$  for 6.8 s. What is the final velocity in km/h?

$$v_i = 30 \text{ km/h} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = +8.3 \text{ m/s}$$

$$a = +3.5 \text{ m/s}^2$$

$$t = 6.8 \text{ s}$$

$$v_f = v_i + at = 8.3 \text{ m/s} + (3.5 \text{ m/s}^2)(6.8 \text{ s}) = 32 \text{ m/s}$$

$$32 \text{ m/s} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 116 \text{ km/h} = \boxed{120 \text{ m/s}} \text{ (2 sf)}$$

3. If a car accelerates from rest at a constant  $5.5 \text{ m/s}^2$ , how long will be required to reach 28 m/s?

$$v_i = 0 \text{ m/s}$$

$$a = +5.5 \text{ m/s}^2$$

$$v_f = 28 \text{ m/s}$$

$$t = ?$$

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{28 \text{ m/s} - 0 \text{ m/s}}{5.5 \text{ m/s}^2} = \boxed{5.1 \text{ s}}$$

4. A car slows from 22 m/s to 3 m/s with a constant acceleration of  $-2.1 \text{ m/s}^2$ . How long does it require?

$$v_i = 22 \text{ m/s}$$

$$v_f = 3 \text{ m/s}$$

$$a = -2.1 \text{ m/s}^2$$

$$t = ?$$

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{3 \text{ m/s} - 22 \text{ m/s}}{-2.1 \text{ m/s}^2} = \boxed{9.0 \text{ s}}$$

$$d = 1/2 (v_f + v_i)t$$

5. A race car traveling at +44 m/s is uniformly accelerated to a velocity of +22 m/s over an 11 s interval. What is its displacement during this time?

$$\begin{aligned} v_i &= +44 \text{ m/s} \\ v_f &= +22 \text{ m/s} \\ t &= 11 \text{ s} \\ d &=? \end{aligned} \quad \begin{aligned} d &= \left( \frac{v_f + v_i}{2} \right) t \\ &= \left( \frac{44 \text{ m/s} + 22 \text{ m/s}}{2} \right) (11 \text{ s}) \\ &= 363 \text{ m} = \underline{360 \text{ m}} \text{ (2 s.f.)} \end{aligned}$$

6. A rocket traveling at +88 m/s is accelerated uniformly to +132 m/s over a 15 s interval. What is its displacement during this time?

$$\begin{aligned} v_i &= +88 \text{ m/s} \\ v_f &= +132 \text{ m/s} \\ t &= 15 \text{ s} \\ d &=? \end{aligned} \quad \begin{aligned} d &= \left( \frac{v_f + v_i}{2} \right) t \\ &= \left( \frac{+88 \text{ m/s} + 132 \text{ m/s}}{2} \right) (15 \text{ s}) \\ &= 1650 \text{ m} \\ &= \underline{1.7 \times 10^3 \text{ m}} \text{ (2 s.f.)} \end{aligned}$$

7. A car accelerates at a constant rate from 15 m/s to 25 m/s while it travels 125 m. How long does this motion take?

$$\begin{aligned} v_i &= 15 \text{ m/s} \\ v_f &= 25 \text{ m/s} \\ d &= 125 \text{ m} \\ t &=? \end{aligned} \quad \begin{aligned} d &= \left( \frac{v_f + v_i}{2} \right) t \\ t &= \frac{2d}{v_f + v_i} \end{aligned} \quad \begin{aligned} t &= \frac{(2)(125 \text{ m})}{(25 \text{ m/s} + 15 \text{ m/s})} \\ &= 6.25 \text{ s} \\ &= \underline{6.3 \text{ s}} \text{ (2 s.f.)} \end{aligned}$$

8. A bike rider accelerates constantly to a velocity of 7.5 m/s during 4.5 s. The bike's displacement is +19 m. What was the initial velocity of the bike?

$$\begin{aligned} v_f &= 7.5 \text{ m/s} \\ t &= 4.5 \text{ s} \\ d &= +19 \text{ m} \\ v_i &=? \end{aligned} \quad \begin{aligned} d &= \left( \frac{v_f + v_i}{2} \right) t \\ v_i &= \frac{2d}{t} - v_f \\ &= \frac{(2)(+19 \text{ m})}{4.5 \text{ s}} - 7.5 \text{ m/s} \\ &= +0.9 \text{ m/s} \text{ (1 s.f. - precision to the nearest tenth of a m/s)} \end{aligned}$$

$$d = v_i t + \frac{1}{2} a t^2$$

9. An airplane starts from rest and accelerates at a constant  $+3.00 \text{ m/s}^2$  for  $30.0 \text{ s}$  before leaving the ground. What is its displacement during this time?

$$\begin{aligned} v_i &= 0 \text{ m/s} \\ a &= +3.00 \text{ m/s}^2 \\ t &= 30.0 \text{ s} \\ d &=? \end{aligned} \quad d = v_i t + \frac{1}{2} a t^2$$

$$= (0 \text{ m/s})(30.0 \text{ m/s}) + \frac{1}{2}(3.00 \text{ m/s}^2)(30.0 \text{ s})^2$$

$$= +1350 \text{ m} \quad (3 \text{ s.f.})$$

10. Starting from rest, a race car moves  $110 \text{ m}$  in the first  $5.0 \text{ s}$  of uniform acceleration.

What is the car's acceleration?

$$\begin{aligned} v_i &= 0 \text{ m/s} \\ d &= +110 \text{ m} \\ t &= 5.0 \text{ s} \\ a &=? \end{aligned} \quad d = \sqrt{v_f^2 + 1/2 a t^2}$$

$$d = \frac{1}{2} a t^2$$

$$a = \frac{2d}{t^2}$$

$$a = \frac{2(+110 \text{ m})}{(5.0 \text{ s})^2}$$

$$= +8.8 \text{ m/s}^2$$

11. A driver brings a car traveling at  $+22 \text{ m/s}$  to a full stop in  $2.0 \text{ s}$ . Assume its acceleration is constant.

a. What is the car's acceleration?

$$\begin{aligned} v_i &= +22 \text{ m/s} \\ v_f &= 0 \text{ m/s} \\ t &= 2.0 \text{ s} \\ a &=? \end{aligned} \quad a = \frac{v_f - v_i}{t}$$

$$= \frac{0 \text{ m/s} - 22 \text{ m/s}}{2.0 \text{ s}}$$

$$= -11 \text{ m/s}^2$$

b. How far does it travel before stopping?

$$\begin{aligned} v_i &= +22 \text{ m/s} \\ v_f &= 0 \text{ m/s} \\ t &= 2.0 \text{ s} \\ d &=? \end{aligned} \quad d = \left( \frac{v_f + v_i}{2} \right) t$$

$$= \left( \frac{+0 \text{ m/s} + 22 \text{ m/s}}{2} \right) (2.0 \text{ s}) = +22 \text{ m}$$

12. A biker passes a lamppost at the crest of a hill at  $+4.5 \text{ m/s}$ . She accelerates down the hill at a constant rate of  $+0.40 \text{ m/s}^2$  for  $12 \text{ s}$ . How far does she move down the hill during this time?

$$\begin{aligned} v_i &= +4.5 \text{ m/s} \\ a &= +0.40 \text{ m/s}^2 \\ t &= 12 \text{ s} \\ d &=? \end{aligned} \quad d = v_i t + \frac{1}{2} a t^2$$

$$= (+4.5 \text{ m/s})(12 \text{ s}) + \frac{1}{2}(+0.40 \text{ m/s}^2)(12 \text{ s})^2$$

$$= 54 \text{ m} + 28.8 \text{ m}$$

$$= 82.8 \text{ m}$$

$$= 83 \text{ m} \quad (2 \text{ s.f.})$$



$$v_f^2 = v_i^2 + 2ad$$

13. An airplane accelerates from a velocity of 21 m/s at the constant rate of 3.0 m/s<sup>2</sup> over +535 m. What is its final velocity?

$$\begin{aligned} v_i &= 21 \text{ m/s} \\ a &= 3.0 \text{ m/s}^2 \\ d &= +535 \text{ m} \\ v_f &=? \end{aligned}$$

$$v_f^2 = v_i^2 + 2ad$$

$$v_f = \sqrt{v_i^2 + 2ad}$$

$$= \sqrt{(21 \text{ m/s})^2 + 2(3.0 \text{ m/s}^2)(535 \text{ m})} = \sqrt{3651 \text{ m}^2/\text{s}^2} = 60.4 \text{ m/s} = 6.0 \times 10^1 \text{ m/s} \quad (\text{2 s.f.})$$

14. The pilot stops the same plane in 484 m using a constant acceleration of -8.0 m/s<sup>2</sup>. How fast was the plane moving before braking began?

$$\begin{aligned} d &= 484 \text{ m} \\ v_f &= 0 \text{ m/s} \\ a &= -8.0 \text{ m/s}^2 \\ v_i &=? \end{aligned}$$

$$v_f^2 = v_i^2 + 2ad$$

$$v_i = \sqrt{v_f^2 - 2ad}$$

$$\begin{aligned} &= \sqrt{0 - (2)(-8.0 \text{ m/s}^2)(484 \text{ m})} \\ &= \sqrt{7744 \text{ m}^2/\text{s}^2} = 88 \text{ m/s} \end{aligned}$$

15. A person wearing a shoulder harness can survive a car crash if the acceleration is smaller than -300.0 m/s<sup>2</sup>. Assuming constant acceleration, how far must the end of the car collapse if it crashes while going 101 km/h?

$$a < -300.0 \text{ m/s}^2$$

$$\begin{aligned} v_i &= 101 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ &= 28.1 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_f &= 0 \text{ m/s} \\ d &=? \end{aligned}$$

$$v_f^2 = v_i^2 + 2ad$$

$$d = \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{(0 \text{ m/s})^2 - (28.1 \text{ m/s})^2}{(2)(-300 \text{ m/s}^2)} = 1.32 \text{ m}$$

16. A car is initially sliding backwards down a hill at -25 km/h. The driver guns the car. By the time the car's velocity is +35 km/h, it is +3.2 m from its starting point. Assuming the car was uniformly accelerated, find the acceleration.

$$\begin{aligned} v_i &= -25 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ &= -6.94 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_f &= +35 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ &= +9.72 \text{ m/s} \end{aligned}$$

$$d = +3.2 \text{ m}$$

$$v_f^2 = v_i^2 + 2ad$$

$$a = \frac{v_f^2 - v_i^2}{2d}$$

$$= \frac{(9.72 \text{ m/s})^2 - (-6.94 \text{ m/s})^2}{(2)(3.2 \text{ m})}$$

$$= +7.2 \text{ m/s}^2$$

- 1a) +1.0m/s b) -1.0m/s 2) 120 km/h 3) 5.1s 4) 9.0s 5) 3.6x10<sup>2</sup>m 6) 1.7x10<sup>3</sup>m 7) 6.3s 8) +0.9m/s 9) 1350m 10) +8.8m/s<sup>2</sup>  
11a) -11m/s<sup>2</sup> b) 22m 12) 83m 13) 6.0x10<sup>1</sup>m/s 14) 88m/s 15) 1.32m 16) +7.2m/s<sup>2</sup>

## 2.4 Acceleration Due to Gravity

- Galileo showed that all objects fall to earth with a constant acceleration, if air resistance can be ignored.
- $g$  is the symbol for acceleration due to gravity.
- on the surface of the earth,  $g = \underline{-9.81} \text{ m/s}^2$  ← Assuming down is negative!
- (varies slightly, depending on distance from centre of earth)
- assuming no air resistance, all acceleration formulas apply to gravity (substitute  $g$  for  $a$ ).

Ex#1: The "Hellavator" ride at Playland falls freely for a time of 1.8 s.

a) What is the velocity at the end of the drop?

$$\begin{aligned}
 v_i &= 0 \text{ m/s} \\
 a &= -9.81 \text{ m/s}^2 \\
 t &= 1.8 \text{ s} \\
 v_f &=? \\
 v_f &= v_i + at \\
 &= 0 \text{ m/s} + (-9.81 \text{ m/s}^2)(1.8 \text{ s}) \\
 &= -17.7 \text{ m/s} = \underline{-18 \text{ m/s}} \text{ 2 s.f.}
 \end{aligned}$$

b) How far does it fall?

$$\begin{aligned}
 v_i &= 0 \text{ m/s} \\
 a &= -9.81 \text{ m/s}^2 \\
 t &= 1.8 \text{ s} \\
 d &=? \\
 d &= v_i t + \frac{1}{2} a t^2 \\
 &= (0 \text{ m/s})(1.8 \text{ s}) + \frac{1}{2} (-9.81 \text{ m/s}^2)(1.8 \text{ s})^2 \\
 &= 0 \text{ m} + (-15.9 \text{ m}) \\
 &= \underline{-16 \text{ m}} \text{ It fell 16m}
 \end{aligned}$$

Ex#2: A ball is thrown straight upwards with an initial velocity of +12 m/s.

a) What is the ball's velocity at the top of its path?

$$v_f = 0 \text{ m/s (at top)}$$

b) how high does the ball go?

$$\begin{aligned}
 v_i &= +12 \text{ m/s} \\
 v_f &= 0 \text{ m/s} \\
 a &= -9.81 \text{ m/s}^2 \\
 d &=? \\
 d &= \frac{v_f^2 - v_i^2}{2a} = \frac{(0 \text{ m/s})^2 - (12 \text{ m/s})^2}{(2)(-9.81 \text{ m/s}^2)} \\
 &= \underline{+7.3 \text{ m}}
 \end{aligned}$$

c) What is the ball's final velocity if it is caught at the same height it was thrown?

$$\begin{aligned}
 v_i &= 0 \text{ m/s} \\
 a &= -9.81 \text{ m/s}^2 \\
 d &= -7.3 \text{ m} \\
 v_f &=? \\
 v_f^2 &= v_i^2 + 2ad \\
 v_f &= \sqrt{v_i^2 + 2ad} \\
 v_f &= \sqrt{(0 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(-7.3 \text{ m})} \\
 &= \underline{-12 \text{ m/s}}
 \end{aligned}$$

d) How long is the ball in the air?

$$\begin{aligned}
 v_i &= +12 \text{ m/s} \\
 v_f &= -12 \text{ m/s} \\
 a &= -9.81 \text{ m/s}^2 \\
 t &=? \\
 v_f &= v_i + at \\
 t &= \frac{v_f - v_i}{a} \\
 t &= \frac{(-12 \text{ m/s}) - (+12 \text{ m/s})}{-9.81 \text{ m/s}^2} \\
 &= \underline{+2.4 \text{ s}}
 \end{aligned}$$